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IS THE SERIAL-POSITION CURVE INVARIANT?

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The evidence presented by McCrary & Hunter and later researchers, which suggests that the form of the serial-position curve is constant throughout variations in a large number of factors that affect rate of learning, has been found inadequate to answer the question of the curve's invariance. Differences in percentage of total errors in serial learning data obtained under different experimental conditions would lead to the conclusion that the serial-position curve is *not* invariant under all the conditions for which invariance has been claimed.

Serial learning data may be analysed into three components—*difficulty* (number of trials to criterion), *efficiency* (percentage of errors), and *relative difficulty* of learning each position, i.e. the so-called serial-position effect. The main methodological inadequacy in studies of the serial-position effect is that the serial-position curve has always been confounded with either one or two of the other components, making it impossible properly to compare various serial curves based on data which differ either in *difficulty* or in *efficiency*. Plotting the serial curves simply in terms of the percentage of total errors at each position does not solve the problem; this method tends to eliminate the *difficulty* component but still leaves the curves confounded with the *efficiency* component.

An index of Relative Difficulty was proposed as a method of representing the serial-position effect. It has the advantages of not being confounded with the other components and of being the same shape whether it is based on errors or on correct responses. It results in what might be called a 'pure' serial-position curve and permits the direct comparison of serial-position curves obtained under various conditions. The Index is recommended as the only satisfactory method for comparing different serial-position curves. It is especially important in comparing serial-position effects in sets of data having different total error percentages. It is urged that future research on the serial-position effect adopt the proposed Index.

I. Introduction

In scientific endeavour the discovery of a constant or an invariant function is always cause for joy. This must be especially true in psychology, where invariance is the rarest of findings. It is worthy of note, therefore, that in recent years the bow-shaped serial-position curve has seemed to be on its way to attaining the status of an invariant function. This interesting development deserves further looking into.

Since at least as far back as 1875, when Ebbinghaus began memorizing lists of nonsense syllables, it has been noted that the beginning and end of the serial list are learned most easily and the middle items are learned with greatest difficulty. If a list of items is learned to mastery and the number of errors made in the course of learning is plotted for each item in the series according to its position, the usual serial-position curve is obtained. It is almost always a skewed, bowed curve, with the maximum errors just past the middle of the list and with the last half of the list having more errors than the first half. This general phenomenon apparently emerges in serial rote-learning regardless of the nature of the items, which may be meaningful words, letters, nonsense syllables, proper names, objects, pictures, figures, etc.

A number of theories have been proposed to account for this so-called 'serial-position effect' (see McGeoch & Irion, 1952, pp. 125–34). According to various theories, the shape of the serial-position curve, particularly the degree of bowing, should be a function of such variables as distribution of practice, rate of presentation of the items, inter-item interval, degree of familiarity with the items, degree of similarity among the items, and individual differences in learning ability. And, indeed, experiments on serial learning have shown that when the serial-position curve is plotted for groups of subjects in terms of the mean number of errors made at each serial position in the course of learning, the serial-position curve does appear to be a function of the above-mentioned variables.

Then McCrary & Hunter (1953) made an interesting discovery which seems to have become an important part of our general knowledge concerning serial learning (e.g. Woodworth & Schlosberg, 1954, p. 713). In essence, McCrary & Hunter found that when the serial-position curve is plotted, not in terms of the mean number of errors made at each position, but in terms of the percentage of the total errors that occurs at each position, none of the variables mentioned above had any effect on the shape of the serial-position curve. The McCrary & Hunter method of plotting the serial curve in terms of the percentage of total errors at each position, which thereby equates the area under all curves, seems reasonable if our interest is in the *shape* of the serial curve rather than in its absolute position on the ordinate. And if theories of serial learning make predictions about the relative difficulty of learning the items at different positions, it would seem reasonable that the errors at each position should be presented relative to the total number of errors that occurred. The percentage method of McCrary & Hunter accomplished this, and therefore should be more suitable than the method of mean errors for testing certain theoretical predictions concerning the *shape* of the serial curve. For example, Hull's theory of serial rote-learning (Hull *et al.* 1940) predicts that massed practice should produce a more bowed serial-position curve than distributed practice, because of a presumably greater accumulation of inhibitory potential in the middle of the list under massed practice. Hovland (1938) plotted serial-position curves for nonsense syllable learning under massed and distributed practice and under 2 and 4 sec. rates of syllable presentation. The curves, plotted as mean errors at each position, strikingly bear out Hull's prediction. But when McCrary & Hunter (1953) plotted these same curves on a percentage basis, the curves became practically identical. McCrary & Hunter showed that the manipulation of other independent variables also produced curves which differ greatly in shape when plotted as mean errors at each position but which assume almost identical shapes when plotted as percentage of errors. Braun & Heymann (1958) went further into this matter by performing an experiment which showed that the error curves (plotted on a percentage basis) for high- and low-meaningful lists of nonsense syllables did not differ. Nor did variations in inter-item interval or inter-trial interval have any effect on the shape of the curve in their study. Thus it would appear from these findings that we have possibly come upon our first 'constant' or 'invariant function' in the psychology of learning.

Indeed, such is the interpretation put upon this finding by some writers. On the basis of these data, for example, Murdock suggested that '...it would almost seem that the shape of the serial-position curve has nothing whatsoever to do with learning' (1960, p. 24) since it seems to be unaffected by variables that are known to affect rate of learning. The invariance of the serial-position curve is essential to Murdock's theory explaining the shape of the curve in terms of differences in the relative distinctiveness of the items in the serial list.

II. A FALLACY

Actually, from the evidence in the literature, we are unable to determine whether or not the serial-position curve is invariant. The analyses of McCrary & Hunter (1953) and of Braun & Heymann (1958) are quite inadequate to answer this question. It is highly probable, however, from evidence in the Braun & Heymann study, that the serial-position curve is *not* invariant. That the idea of invariance even arose is mainly the result of a faulty analysis of what the serial-position curve is actually supposed to represent. It has apparently been assumed that there is just *one* serial-position curve for a given set of data, viz. the error curve, plotted either in terms of mean errors at each position or in terms of the percentage of total errors at each position. But the serial curve can also be expressed in terms of the number (or percentage) of *correct* responses at each position. It is sometimes plotted in this manner (e.g. McGeoch & Irion, 1952, p. 116). The important point, however, is that the *error* curve and the *correct* curve (whether in terms of absolute number or of percentage) for the same set of data are not always the same shape It is possible for one curve to be much more bowed than the other. We cannot determine from the data in the McCrary & Hunter study how 'constant' the serial-position curves would appear if they had been plotted in terms of *correct* responses rather than in terms of *errors*.

Some contrived curves can illustrate the possible fallacy in the McCrary & Hunter analysis. Fig. 1 shows two fictitious but typical serial-position curves, *A* and *B*, plotted in terms of number of errors at each position. Curve *B* appears much more bowed than curve *A*. When these two curves are each expressed as the percentage of total errors at each position, they become *identical*, as shown in Fig. 2. This is what happened to all the curves in the McCrary & Hunter and the Braun & Heymann studies. But now look at the curves for *percentage correct* in Fig. 2. Plotted on this basis the curves are quite different in shape, curve *B* being much more bowed than curve *A*. The reason this happens is that curves *A* and *B* are each based on a different total error percentage. Curve *A* had 28·75 % errors. Curve *B* had 57·50 % errors.

If two or more percentage *error* curves are of the same shape but are based on different percentages or proportions of total errors (i.e. total errors/total responses) it is certain that the percentage *correct* curves are not of the same relative shapes. The percentage *error* curves in the Braun & Heymann study are all practically identical, but since they are based on quite different total error percentages, varying from 39 to 59 %, it is impossible that their data, when plotted in terms of percentage correct at each position, could yield identical curves for the various experimental conditions they investigated. Unfortunately the percentage correct curves cannot be plotted with any exactitude from the data presented by Braun & Heymann. The point can be illustrated, however, by some of the writer's data on serial learning. From a group of sixty college students who had learned a 9-item serial list consisting of coloured geometric forms (triangles, squares, and circles coloured red, blue and yellow) the average serial-position curve of the ten subjects (group A) with the highest error percentage (mean = 57 %) was compared with that of the ten subjects (group B) with the lowest error percentage (mean 33 %). Fig. 3 shows the percentage error curves and the percentage correct curves for the two groups. In this case it is evident that the degree of bowing of the two curves is actually *reversed* for the two methods

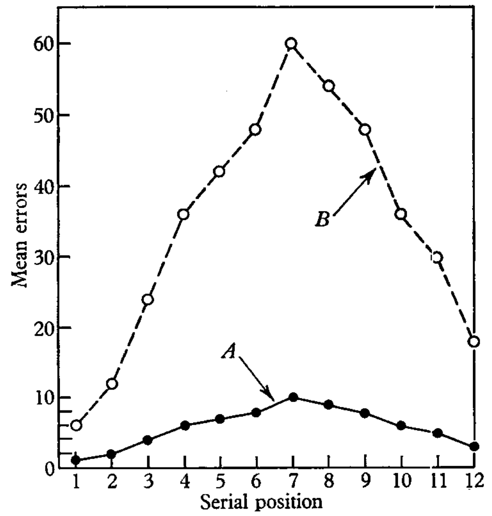

Fig. 1

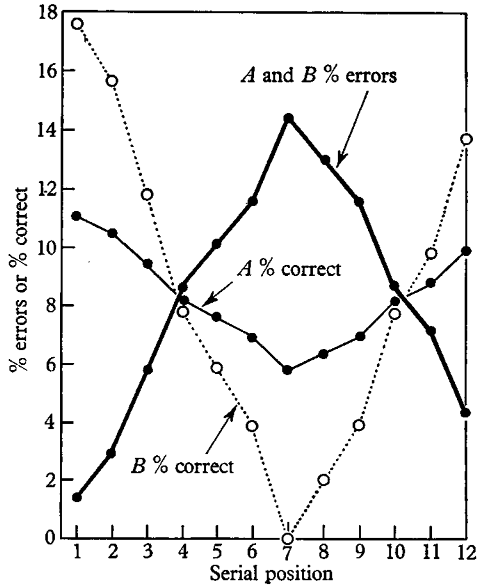

Fig. 2

Fig. 1. Two serial-position curves plotted in terms of number of errors at each position.

Fig. 2. The same 'data' as in Fig. 1, but here plotted in terms of the percentage of total errors at each position and the percentage of total correct at each position.

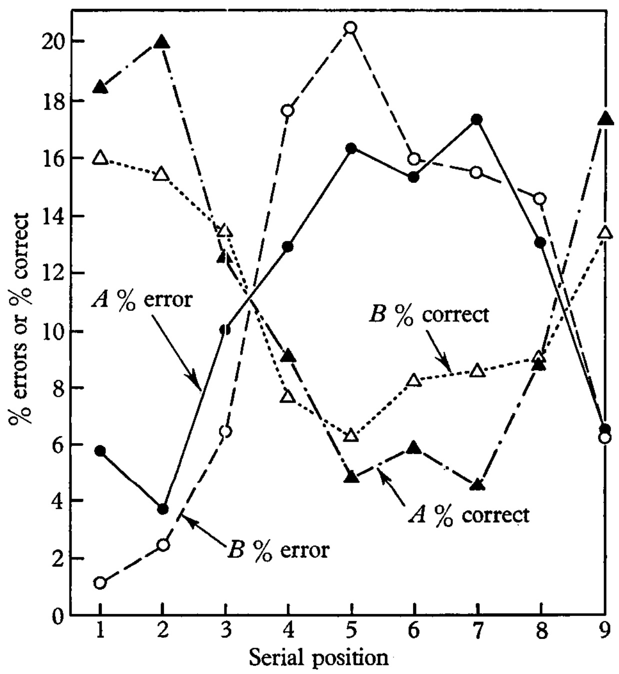

Fig. 3. Serial-position curves plotted in terms of percentage of errors and percentage correct at each position. Group A has a mean total error of 57 %, group B of 33 %.

of plotting curves. Curve *A* shows the least difference in shape for the two methods, because its total error percentage is closer to 50 % than that of group B. Only if both groups had the same error percentage would it be proper to compare their serial-position curves by only one method, i.e. either in terms of percentage of total *errors* or in terms of percentage of total *correct* at each position. To be more exact, one must insist that all subjects in each group have the same total error percentage if their averaged curves are to be properly compared. Without knowledge of these conditions, which most likely are not even approximated in the McCrary & Hunter and the Braun & Heymann studies, along with the absence of percentage correct curves, there is unfortunately no conclusion of any general importance to our understanding of the serial-position effect that can be drawn from these studies, other than the general fact that the serial-position effect is manifested under a variety of conditions.

III. A SOLUTION

Serial rote-learning data may be analysed into three essential components:

(1) The *difficulty* of the learning task, represented by the number of trials or stimulus presentations required to learn to a particular criterion.

(2) The *efficiency* of learning, represented by the percentage of correct responses (or the percentage of errors) during the course of learning to a particular criterion.

(3) The *relative difficulty* of learning the various positions. This actually is the serial-position curve.

Four methods of plotting serial-position curves are to be found in the literature, and all are inadequate in that they confound at least two or more of the above-mentioned components. These methods are applied either to errors or to correct responses, but error curves have been the most frequently used.

The methods are:

(*a*) Mean number of errors at each position. This curve confounds components 1, 2, and 3 above. It also has the disadvantage that subjects are weighted unequally in the means, so that the average curve for the group will tend more to reflect the shape of the curve for slow learners (i.e. those with many errors) than that of the fast learners. For testing theoretical predictions concerning the effects of different parameters on the shape of the serial curve, this method is unsatisfactory. However, it has been the most often used, especially in early studies.

(*b*) Percentage of errors at each position. This is determined by dividing the number of errors for all subjects at each position by the total number of errors for all positions. It has the effect of equating all curves for difficulty, i.e. it makes the area under all curves the same. But the shape of the curve is still confounded by the *efficiency* component. Furthermore, like curve *a*, subjects are weighted unequally in the group curve, those with a large number of errors determining the shape of the curve more than those with few errors. Thus, this method of representing the serial-position effect is also unsatisfactory. This was the method used by McCrary & Hunter (1953) and by Braun & Heymann (1958).

(*c*) Mean percentage of errors at each position. In this method the percentage of errors at each position *for each* subject is determined (by dividing the subject's total errors into the number of errors at each position) and these percentages are then averaged to produce the group curve. This method is correct in weighting every subject equally in the average curve. And like curve *B*, it equates all curves on the difficulty component, making the areas under all curves the same. But the *efficiency* component is still confounded in the curve, which means that the error curve and correct curve will not necessarily have the same shape.

(*d*) Mean logarithm of errors at each position. This method has been used by Glanzer & Peters (1960). Since the work of McCrary & Hunter suggests that the total number of errors has a multiplicative effect on the shape of the serial-position curve, the transformation of the errors into log errors converts the multiplicative effect into an additive factor. This method has essentially the same effect as methods *b* and *c*. When it is applied to the McCrary & Hunter mean error curves, for example, it makes them all the same shape, as does the percentage method, but the curves still have different positions on the ordinate. This logarithmic method, however, is like the percentage method in that the efficiency component is still confounded in the serial curve and it is possible to obtain differently shaped curves for errors and for correct responses on the same set of data. In other words, in using methods *a*, *b*, *c*, or *d* a single curve cannot properly represent the serial-position effect. Comparing serial curves by analysis of variance, of course, does not overcome these shortcomings as long as errors, correct responses, or percentage errors are used in the analysis of variance.

Ideally, each of the three components of serial learning data should be kept separate from the others. What we really wish to know when we speak of the serial-position effect or the shape of the serial-position curve is the *relative difficulty* of learning each position, unconfounded by the absolute difficulty of the task (as measured by total trials) or the *efficiency* of learning (as measured by the total percentage of errors). Method *a* confounds all three variables, and methods *b*, *c* and *d* confound the *relative difficulty* and the *efficiency* components. If these components covaried perfectly, there would be no problem. But they do not. For example, in a group of sixty college students who had learned a 9-item serial list (coloured geometric forms) by the anticipation method (3 sec. rate of presentation, 6 sec. inter-trial interval) the following intercorrelations were obtained.

$r = 0{\cdot}92$ ($P < 0{\cdot}01$) between *number of errors* and *number of trials* to attain the criterion of one perfect trial.

$r = 0{\cdot}45$ ($P < 0{\cdot}01$) between *number of errors* and *percentage of errors*.

$r = 0{\cdot}15$ (N.S.) between *number of trials* and *percentage of errors*.

There not only appear to be reliable individual differences in each of the components, but examination of the literature reveals that independent variables which affect the difficulty of the task (e.g. distribution of practice, rate of presentation, degree of intralist similarity, familiarity and meaningfulness of the items) also affect the efficiency of learning, the more difficult tasks having a higher error percentage.

The Index of Relative Difficulty

In order that the serial-position curve shall represent what we actually mean by the serial-position effect, i.e. the *relative difficulty* of learning each position, unconfounded by the difficulty of the task or the efficiency of learning, an Index of Relative

Difficulty is here proposed as the only entirely satisfactory method of representing the serial-position effect. It has the advantage that it shows the *shape* of the serial curve independently of difficulty and efficiency. Also, the curves based on errors and on correct responses are always exactly the same shape, one being simply the upside-down mirror image of the other. This method permits valid direct comparisons of serial curves obtained under any conditions.

So that the serial curve of each subject is equally weighted in the average curve for the group, the Index is applied to the data for each subject. The points on the curve are then converted to percentages for each subject and are averaged over all subjects to produce the group curve.

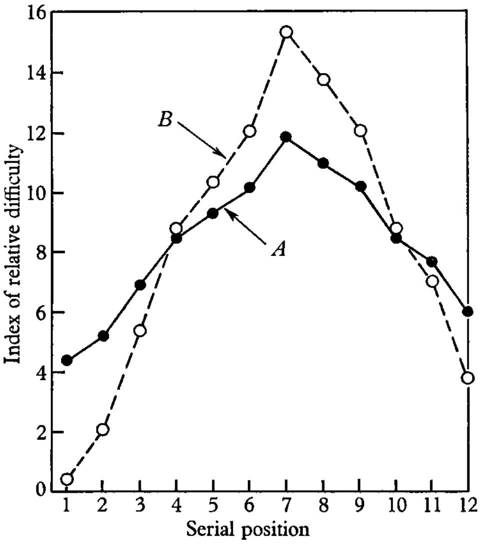

Fig. 4

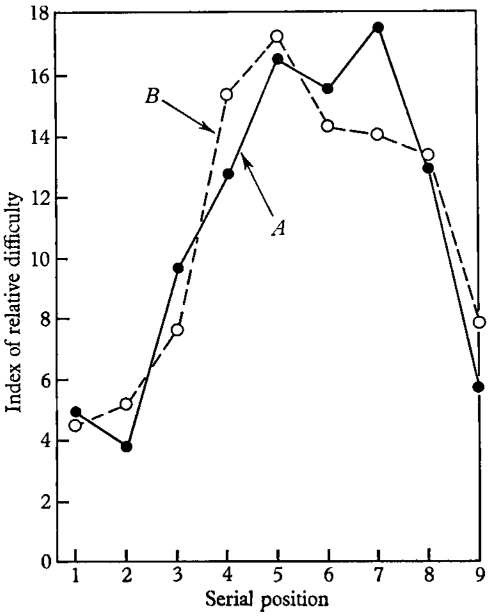

Fig. 5

Fig. 4. The data of Fig. 1 presented in the form of the Index of Relative Difficulty.

Fig. 5. The data of Fig. 3 presented in the form of the Index of Relative Difficulty.

The method of obtaining the Index for a single subject is as follows:

(*a*) *Correction factor*. This, in effect, equates all curves for efficiency, i.e. total error percentage. When the error percentage is 50 %, the error curve and correct curve are of identical shape. Thus the method need only be applied to the errors.

$$\text{Correction factor}^* = \frac{\left(\dfrac{50}{\%\ \text{errors}} \times \text{number of responses}\right) - \text{number of responses}}{\text{Number of positions in series}}.$$

* The % errors = 100 × total number of errors/total responses.

The number of responses = number of trials to criterion × number of positions in series; in other words, the number of opportunities to make a correct response.

In actual computation the following formula, which is algebraically equivalent to that above, should be used:

$$\text{Correction factor} = 1/P(0{\cdot}50\,R - E),$$

where P = number of positions, R = total number of responses, and E = number of errors.

(*b*) Algebraically add the Correction Factor to the number of errors at each position. (Note that the Correction Factor may be either positive or negative.)

(*c*) After performing *b*, obtain the sum total over all positions.

(*d*) Divide *c* into each position, *b*, to convert the quantities into percentages which will sum to 100%.

(*e*) The group curve is obtained by averaging *d* for each position over all subjects. The curve thus obtained is an Index of Relative Difficulty. Fig. 4 shows the same data as in Fig. 1 presented in the form of the Index. If the Index were computed from the correct responses rather than from the errors, the curves would simply be the upside-down mirror images of those in Fig. 4; in other words, they would contain identical information. We see that the two curves (*A* and *B*) in Fig. 4 are actually of different shapes and are *not* identical, as fallaciously represented by the percentage error curves in Fig. 2. From Fig. 5 we are able to make a valid statement concerning the relative degrees of bowing of the two curves, whereas on the basis of the percentage curves in Fig. 3 our statements about the relative degrees of bowing of curves *A* and *B* depend upon whether we are looking at the 'error' curves or the 'correct' curves. But for any one set of data there should be only one curve representing the serial-position effect, i.e. the relative difficulty of learning each position. This curve is provided by the Index of Relative Difficulty. When serial curves are compared by analysis of variance, the analysis should be performed on the Index scores.

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